GUIDED PRACTICE for Example 1

List the possible rational zeros of f using the rational zero theorem.

1. $f(x) = x^3 + 9x^2 + 23x + 15$ **2.** $f(x) = 2x^3 + 3x^2 - 11x - 6$

VERIFYING ZEROS In Lesson 5.5, you found zeros of polynomial functions when one zero was known. The rational zero theorem is a starting point for finding zeros when no zeros are known.

However, the rational zero theorem lists only *possible* zeros. In order to find the *actual* zeros of a polynomial function f, you must test values from the list of possible zeros. You can test a value by evaluating $f(x)$ using the test value as x.

EXAMPLE 2 Find zeros when the leading coefficient is 1

Find all real zeros of $f(x) = x^3 - 8x^2 + 11x + 20$.

Solution

STEP 1 **List** the possible rational zeros. The leading coefficient is 1 and the constant term is 20. So, the possible rational zeros are:

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$$

AVOID ERRORS
Notice that not every *possible* zero generated by the rational zero theorem is an *actual* zero of *f*.

STEP 2 **Test** these zeros using synthetic division.

Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 11 & 20 \\ & & 1 & -7 & 4 \\ \hline & 1 & -7 & 4 & \mathbf{24} \end{array}$$

1 is not a zero.

Test $x = -1$:

$$\begin{array}{r|rrrr} -1 & 1 & -8 & 11 & 20 \\ & & -1 & 9 & -20 \\ \hline & 1 & -9 & 20 & \mathbf{0} \end{array}$$

−1 is a zero.

Because -1 is a zero of f, you can write $f(x) = (x + 1)(x^2 - 9x + 20)$.

STEP 3 **Factor** the trinomial in $f(x)$ and use the factor theorem.

$$f(x) = (x + 1)(x^2 - 9x + 20) = (x + 1)(x - 4)(x - 5)$$

▶ The zeros of f are -1, 4, and 5.

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GUIDED PRACTICE for Example 2

Find all real zeros of the function.

3. $f(x) = x^3 - 4x^2 - 15x + 18$ **4.** $f(x) = x^3 - 8x^2 + 5x + 14$

LIMITING THE SEARCH FOR ZEROS In Example 2, the leading coefficient of the polynomial function is 1. When the leading coefficient is not 1, the list of possible rational zeros can increase dramatically. In such cases, the search can be shortened by sketching the function's graph.