5.6 Find Rational Zeros

TEKS 2A.8.B; P.1.D, P.3.A, P.3.B

Before You found the zeros of a polynomial function given one zero.

Now You will find all real zeros of a polynomial function.

Why? So you can model manufacturing processes, as in Ex. 45.

Key Vocabulary
- **zero of a function,** *p. 254*
- **constant term,** *p. 337*
- **leading coefficient,** *p. 337*

The polynomial function $f(x) = 64x^3 + 152x^2 - 62x - 105$ has $-\frac{5}{2}$, $-\frac{3}{4}$, and $\frac{7}{8}$ as its zeros. Notice that the numerators of these zeros (-5, -3, and 7) are factors of the constant term, -105. Also notice that the denominators (2, 4, and 8) are factors of the leading coefficient, 64. These observations are generalized by the *rational zero theorem*.

KEY CONCEPT — For Your Notebook

The Rational Zero Theorem

If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

EXAMPLE 1 List possible rational zeros

List the possible rational zeros of f using the rational zero theorem.

a. $f(x) = x^3 + 2x^2 - 11x + 12$

Factors of the constant term: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Factors of the leading coefficient: ±1

Possible rational zeros: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

Simplified list of possible zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

b. $f(x) = 4x^4 - x^3 - 3x^2 + 9x - 10$

Factors of the constant term: $\pm1, \pm2, \pm5, \pm10$

Factors of the leading coefficient: $\pm1, \pm2, \pm4$

Possible rational zeros:

$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{5}{2}, \pm\frac{10}{2}, \pm\frac{1}{4}, \pm\frac{2}{4}, \pm\frac{5}{4}, \pm\frac{10}{4}$

Simplified list of possible zeros: $\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{4}, \pm\frac{5}{4}$

AVOID ERRORS
Be sure your lists include both the positive and negative factors of the constant term and the leading coefficient.