**FACTOR THEOREM** Suppose the remainder is 0 when a polynomial $f(x)$ is divided by $x - k$. Then

$$\frac{f(x)}{x-k} = q(x) + \frac{0}{x-k} = q(x)$$

where $q(x)$ is the quotient polynomial. Therefore, $f(x) = (x - k) \cdot q(x)$, so that $x - k$ is a factor of $f(x)$. This result is summarized by the *factor theorem*.

## KEY CONCEPT — *For Your Notebook*

### Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

The factor theorem can be used to solve a variety of problems.

| Problem | Example |
|---|---|
| Given one *factor* of a polynomial, find the other *factors*. | See Example 4 below. |
| Given one *zero* of a polynomial function, find the other *zeros*. | See Example 5 on page 365. |
| Given one *solution* of a polynomial equation, find the other *solutions*. | See Example 6 on page 365. |

## EXAMPLE 4 Factor a polynomial

**Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.**

**Solution**

Because $x + 2$ is a factor of $f(x)$, you know that $f(-2) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} -2 & 3 & -4 & -28 & -16 \\ & & -6 & 20 & 16 \\ \hline & 3 & -10 & -8 & 0 \end{array}$$

Use the result to write $f(x)$ as a product of two factors and then factor completely.

$f(x) = 3x^3 - 4x^2 - 28x - 16$ — **Write original polynomial.**

$= (x + 2)(3x^2 - 10x - 8)$ — **Write as a product of two factors.**

$= (x + 2)(3x + 2)(x - 4)$ — **Factor trinomial.**

**AVOID ERRORS**
The remainder after using synthetic division should always be zero when you are dividing a polynomial by one of its factors.

## ✓ GUIDED PRACTICE for Examples 3 and 4

**Divide using synthetic division.**

**3.** $(x^3 + 4x^2 - x - 1) \div (x + 3)$

**4.** $(4x^3 + x^2 - 3x + 7) \div (x - 1)$

**Factor the polynomial completely given that $x - 4$ is a factor.**

**5.** $f(x) = x^3 - 6x^2 + 5x + 12$

**6.** $f(x) = x^3 - x^2 - 22x + 40$