

EXAMPLE 2 Use polynomial long division with a linear divisor

Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$.

$$\begin{array}{r}
 x^2 + 7x + 7 \leftarrow \text{quotient} \\
 x - 2 \overline{) x^3 + 5x^2 - 7x + 2} \\
 \underline{x^3 - 2x^2} \qquad \qquad \text{Multiply divisor by } x^3/x = x^2. \\
 7x^2 - 7x \qquad \qquad \text{Subtract.} \\
 \underline{7x^2 - 14x} \qquad \qquad \text{Multiply divisor by } 7x^2/x = 7x. \\
 7x + 2 \qquad \qquad \text{Subtract.} \\
 \underline{7x - 14} \qquad \qquad \text{Multiply divisor by } 7x/x = 7. \\
 16 \leftarrow \text{remainder}
 \end{array}$$

$$\blacktriangleright \frac{x^3 + 5x^2 - 7x + 2}{x - 2} = x^2 + 7x + 7 + \frac{16}{x - 2}$$

GUIDED PRACTICE for Examples 1 and 2

Divide using polynomial long division.

- $(2x^4 + x^3 + x - 1) \div (x^2 + 2x - 1)$
- $(x^3 - x^2 + 4x - 10) \div (x + 2)$

SYNTHETIC DIVISION If you use synthetic substitution to evaluate $f(x)$ in Example 2 when $x = 2$, as shown below, you can see that $f(2)$ equals the remainder when $f(x)$ is divided by $x - 2$. Also, the other values below the line match the coefficients of the quotient. For this reason, synthetic substitution is sometimes called **synthetic division**. Synthetic division can be used to divide any polynomial by a divisor of the form $x - k$.

$$\begin{array}{r|rrrr}
 2 & 1 & 5 & -7 & 2 \\
 & & 2 & 14 & 14 \\
 \hline
 & 1 & 7 & 7 & 16
 \end{array}$$

coefficients of quotient \longrightarrow 1 7 7 16 \longleftarrow remainder

KEY CONCEPT

For Your Notebook

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

EXAMPLE 3 Use synthetic division

Divide $f(x) = 2x^3 + x^2 - 8x + 5$ by $x + 3$ using synthetic division.

$$\begin{array}{r|rrrr}
 -3 & 2 & 1 & -8 & 5 \\
 & & -6 & 15 & -21 \\
 \hline
 & 2 & -5 & 7 & -16
 \end{array}$$

$$\blacktriangleright \frac{2x^3 + x^2 - 8x + 5}{x + 3} = 2x^2 - 5x + 7 - \frac{16}{x + 3}$$

DIVIDE POLYNOMIALS

Because the divisor is $x + 3 = x - (-3)$, evaluate the dividend when $x = -3$.