5.5 Apply the Remainder and Factor Theorems

TEKS 2A.2.A; P.1.D, P.3.A, P.3.B

Before You used special patterns to factor polynomials.

Now You will use theorems to factor polynomials.

Why? So you can determine attendance at sports games, as in Ex. 43.

Key Vocabulary
- **polynomial long division**
- **synthetic division**

When you divide a polynomial $f(x)$ by a divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor. One way to divide polynomials is called **polynomial long division**.

EXAMPLE 1 Use polynomial long division

Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$.

Solution

Write polynomial division in the same format you use when dividing numbers. Include a "0" as the coefficient of x^2 in the dividend. At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

$$\begin{array}{rrrrrrl}
 & & & 3x^2 + & 4x - & 3 & \leftarrow \text{quotient} \\
x^2 - 3x + 5 \,\big)\, & 3x^4 - & 5x^3 + & 0x^2 + & 4x - & 6 & \\
 & 3x^4 - & 9x^3 + & 15x^2 & & & \text{Multiply divisor by } 3x^4/x^2 = 3x^2. \\
 & & 4x^3 - & 15x^2 + & 4x & & \text{Subtract. Bring down next term.} \\
 & & 4x^3 - & 12x^2 + & 20x & & \text{Multiply divisor by } 4x^3/x^2 = 4x. \\
 & & & -3x^2 - & 16x - & 6 & \text{Subtract. Bring down next term.} \\
 & & & -3x^2 + & 9x - & 15 & \text{Multiply divisor by } -3x^2/x^2 = -3. \\
 & & & & -25x + & 9 & \leftarrow \text{remainder}
\end{array}$$

AVOID ERRORS
The expression added to the quotient in the result of the long division problem is $\frac{r(x)}{d(x)}$, not $r(x)$.

$$\frac{3x^4 - 5x^3 + 4x - 6}{x^2 - 3x + 5} = 3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}$$

CHECK You can check the result of a division problem by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$(3x^2 + 4x - 3)(x^2 - 3x + 5) + (-25x + 9)$$
$$= 3x^2(x^2 - 3x + 5) + 4x(x^2 - 3x + 5) - 3(x^2 - 3x + 5) - 25x + 9$$
$$= 3x^4 - 9x^3 + 15x^2 + 4x^3 - 12x^2 + 20x - 3x^2 + 9x - 15 - 25x + 9$$
$$= 3x^4 - 5x^3 + 4x - 6 \checkmark$$