FACTORING PATTERNS In part (b) of Example 1, the special factoring pattern for the difference of two squares is used to factor the expression completely. There are also factoring patterns that you can use to factor the sum or difference of two cubes.

## KEY CONCEPT

For Your Notebook

## Special Factoring Patterns

## Sum of Two Cubes

$a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$

## Example

$$
\begin{aligned}
8 x^{3}+27 & =(2 x)^{3}+3^{3} \\
& =(2 x+3)\left(4 x^{2}-6 x+9\right)
\end{aligned}
$$

## Example

$$
\begin{aligned}
64 x^{3}-1 & =(4 x)^{3}-1^{3} \\
& =(4 x-1)\left(16 x^{2}+4 x+1\right)
\end{aligned}
$$

## EXAMPLE 2 Factor the sum or difference of two cubes

## Factor the polynomial completely.

a. $x^{3}+64=x^{3}+4^{3}$

$$
=(x+4)\left(x^{2}-4 x+16\right)
$$

b. $16 z^{5}-250 z^{2}=2 z^{2}\left(8 z^{3}-125\right)$

$$
\begin{aligned}
& =2 z^{2}\left[(2 z)^{3}-5^{3}\right] \\
& =2 z^{2}(2 z-5)\left(4 z^{2}+10 z+25\right)
\end{aligned}
$$

Sum of two cubes

Factor common monomial. Difference of two cubes

## GUIDED Practice for Examples 1 and 2

## Factor the polynomial completely.

1. $x^{3}-7 x^{2}+10 x$
2. $3 y^{5}-75 y^{3}$
3. $16 b^{5}+686 b^{2}$
4. $w^{3}-27$

FACTORING BY GROUPING For some polynomials, you can factor by grouping pairs of terms that have a common monomial factor. The pattern for factoring by grouping is shown below.

$$
\begin{aligned}
r a+r b+s a+s b & =r(a+b)+s(a+b) \\
& =(r+s)(a+b)
\end{aligned}
$$

## EXAMPLE 3 Factor by grouping

## AVOID ERRORS

An expression is not factored completely until all factors, such as $x^{2}-16$, cannot be factored further.

Factor the polynomial $x^{3}-3 x^{2}-16 x+48$ completely.

$$
\begin{aligned}
x^{3}-3 x^{2}-16 x+48 & =x^{2}(x-3)-16(x-3) & & \text { Factor by grouping. } \\
& =\left(x^{2}-16\right)(x-3) & & \text { Distributive property } \\
& =(x+4)(x-4)(x-3) & & \text { Difference of two squares }
\end{aligned}
$$

