EXAMPLE 2  Evaluate by direct substitution

Use direct substitution to evaluate \( f(x) = 2x^4 - 5x^3 - 4x + 8 \) when \( x = 3 \).

\[
\begin{align*}
  f(x) &= 2x^4 - 5x^3 - 4x + 8 \quad \text{Write original function.} \\
  f(3) &= 2(3)^4 - 5(3)^3 - 4(3) + 8 \quad \text{Substitute 3 for } x. \\
  &= 162 - 135 - 12 + 8 \quad \text{Evaluate powers and multiply.} \\
  &= 23 \quad \text{Simplify.}
\end{align*}
\]

✓ GUIDED PRACTICE for Examples 1 and 2

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

1. \( f(x) = 13 - 2x \)
2. \( p(x) = 9x^4 - 5x^2 + 4 \)
3. \( h(x) = 6x^2 + \pi - 3x \)

Use direct substitution to evaluate the polynomial function for the given value of \( x \).

4. \( f(x) = x^4 + 2x^3 + 3x^2 - 7; x = -2 \)
5. \( g(x) = x^3 - 5x^2 + 6x + 1; x = 4 \)

SYNTHETIC SUBSTITUTION  Another way to evaluate a polynomial function is to use **synthetic substitution**. This method, shown in the next example, involves fewer operations than direct substitution.

EXAMPLE 3  Evaluate by synthetic substitution

Use synthetic substitution to evaluate \( f(x) \) from Example 2 when \( x = 3 \).

Solution

\[
\begin{align*}
  \text{STEP 1} & \quad \text{Write the coefficients of } f(x) \text{ in order of descending exponents. Write the value at which } f(x) \text{ is being evaluated to the left.} \\
  & \quad \begin{array}{rrrrr}
  \text{x-value} & 3 & 2 & -5 & 0 & -4 & 8 \\
  \text{coefficients} & & & & & & \\
  \end{array} \\
  \text{STEP 2} & \quad \text{Bring down the leading coefficient. Multiply the leading coefficient by the } x\text{-value. Write the product under the second coefficient. Add.} \\
  & \quad \begin{array}{rrrrr}
  & 3 & 2 & -5 & 0 & -4 & 8 \\
  & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
  & 6 & 1 \\
  \end{array} \\
  \text{STEP 3} & \quad \text{Multiply the previous sum by the } x\text{-value. Write the product under the third coefficient. Add. Repeat for all of the remaining coefficients. The final sum is the value of } f(x) \text{ at the given } x\text{-value.} \\
  & \quad \begin{array}{rrrrr}
  & 3 & 2 & -5 & 0 & -4 & 8 \\
  & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
  & 6 & 3 & 9 & 15 & \downarrow \\
  & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
  & 23 \\
  \end{array} \\
\end{align*}
\]

Synthetic substitution gives \( f(3) = 23 \), which matches the result in Example 2.