5.2 Evaluate and Graph Polynomial Functions

2A.4.B; P.1.E, P.3.A, P.3.B

Before	You evaluated and graphed linear and quadratic functions.
Now	You will evaluate and graph other polynomial functions.
Why?	So you can model skateboarding participation, as in Ex. 55.

Key Vocabulary
- polynomial
- polynomial function
- synthetic substitution
- end behavior

Recall that a monomial is a number, a variable, or a product of numbers and variables. A **polynomial** is a monomial or a sum of monomials. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. For this function, a_n is the **leading coefficient**, n is the **degree**, and a_0 is the **constant term**. A polynomial function is in **standard form** if its terms are written in descending order of exponents from left to right.

Common Polynomial Functions

Degree	Type	Standard form	Example
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1 x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2 x^2 + a_1 x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^4 + 2x - 1$

EXAMPLE 1 Identify polynomial functions

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a. $h(x) = x^4 - \frac{1}{4}x^2 + 3$

b. $g(x) = 7x - \sqrt{3} + \pi x^2$

c. $f(x) = 5x^2 + 3x^{-1} - x$

d. $k(x) = x + 2^x - 0.6x^5$

Solution

a. The function is a polynomial function that is already written in standard form. It has degree 4 (quartic) and a leading coefficient of 1.

b. The function is a polynomial function written as $g(x) = \pi x^2 + 7x - \sqrt{3}$ in standard form. It has degree 2 (quadratic) and a leading coefficient of π.

c. The function is not a polynomial function because the term $3x^{-1}$ has an exponent that is not a whole number.

d. The function is not a polynomial function because the term 2^x does not have a variable base and an exponent that is a whole number.