## EXAMPLES

6 and 7
on p. 287
for Exs. 41-49

FINDING THE VERTEX In Exercises 39 and 40, use completing the square to find the vertex of the given function's graph. Then tell what the vertex represents.
39. At Buckingham Fountain in Chicago, the water's height $h$ (in feet) above the main nozzle can be modeled by $h=-16 t^{2}+89.6 t$ where $t$ is the time (in seconds) since the water has left the nozzle.
40. When you walk $x$ meters per minute, your rate $y$ of energy use (in calories per minute) can be modeled by $y=0.0085 x^{2}-1.5 x+120$.


Buckingham Fountain

WRITING IN VERTEX FORM Write the quadratic function in vertex form. Then identify the vertex.
41. $y=x^{2}-8 x+19$
42. $y=x^{2}-4 x-1$
43. $y=x^{2}+12 x+37$
44. $y=x^{2}+20 x+90$
(45.) $f(x)=x^{2}-3 x+4$
46. $g(x)=x^{2}+7 x+2$
47. $y=2 x^{2}+24 x+25$
48. $y=5 x^{2}+10 x+7$
49. $y=2 x^{2}-28 x+99$

ERROR ANALYSIS Describe and correct the error in solving the equation.
50.

$$
\begin{aligned}
x^{2}+10 x+13 & =0 \\
x^{2}+10 x & =-13 \\
x^{2}+10 x+25 & =-13+25 \\
(x+5)^{2} & =12 \\
x+5 & = \pm \sqrt{12} \\
x & =-5 \pm \sqrt{12} \\
x & =-5 \pm 4 \sqrt{3}
\end{aligned}
$$

51. 

$$
\begin{aligned}
4 x^{2}+24 x-11 & =0 \\
4\left(x^{2}+6 x\right) & =11 \\
4\left(x^{2}+6 x+9\right) & =11+9 \\
4(x+3)^{2} & =20 \\
(x+3)^{2} & =5 \\
x+3 & = \pm \sqrt{5} \\
x & =-3 \pm \sqrt{5}
\end{aligned}
$$

COMPLETING THE SQUARE Solve the equation by completing the square.
52. $x^{2}+9 x+20=0$
53. $x^{2}+3 x+14=0$
54. $7 q^{2}+10 q=2 q^{2}+155$
55. $3 x^{2}+x=2 x-6$
56. $0.1 x^{2}-x+9=0.2 x$
57. $0.4 v^{2}+0.7 v=0.3 v-2$
58. TAKS REASONING Write a quadratic equation with real-number solutions that can be solved by completing the square but not by factoring.
59. TAKS REASONING In this exercise, you will investigate the graphical effect of completing the square.
a. Graph each pair of functions in the same coordinate plane.

$$
\begin{array}{lll}
y=x^{2}+2 x & y=x^{2}+4 x & y=x^{2}-6 x \\
y=(x+1)^{2} & y=(x+2)^{2} & y=(x-3)^{2}
\end{array}
$$

b. Compare the graphs of $y=x^{2}+b x$ and $y=\left(x+\frac{b}{2}\right)^{2}$. What happens to the graph of $y=x^{2}+b x$ when you complete the square?
60. REASONING For what value(s) of $k$ does $x^{2}+b x+\left(\frac{b}{2}\right)^{2}=k$ have exactly 1 real solution? 2 real solutions? 2 imaginary solutions?
61. CHALLENGE Solve $x^{2}+b x+c=0$ by completing the square. Your answer will be an expression for $x$ in terms of $b$ and $c$.

