EXAMPLE 2 Make a perfect square trinomial

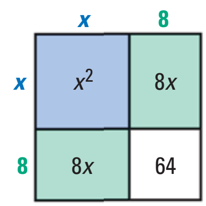
Find the value of c that makes $x^2 + 16x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

Solution

STEP 1 **Find** half the coefficient of x. $\frac{16}{2} = 8$

STEP 2 **Square** the result of Step 1. $8^2 = 64$

STEP 3 **Replace** c with the result of Step 2. $x^2 + 16x + 64$



▶ The trinomial $x^2 + 16x + c$ is a perfect square when $c = 64$. Then $x^2 + 16x + 64 = (x + 8)(x + 8) = (x + 8)^2$.

GUIDED PRACTICE for Examples 1 and 2

Solve the equation by finding square roots.

1. $x^2 + 6x + 9 = 36$ **2.** $x^2 - 10x + 25 = 1$ **3.** $x^2 - 24x + 144 = 100$

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

4. $x^2 + 14x + c$ **5.** $x^2 + 22x + c$ **6.** $x^2 - 9x + c$

SOLVING EQUATIONS The method of completing the square can be used to solve *any* quadratic equation. When you complete a square as part of solving an equation, you must add the same number to *both* sides of the equation.

EXAMPLE 3 Solve $ax^2 + bx + c = 0$ when $a = 1$

Solve $x^2 - 12x + 4 = 0$ by completing the square.

$x^2 - 12x + 4 = 0$ Write original equation.

$x^2 - 12x = -4$ Write left side in the form $x^2 + bx$.

$x^2 - 12x + 36 = -4 + 36$ Add $\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$ to each side.

$(x - 6)^2 = 32$ Write left side as a binomial squared.

$x - 6 = \pm\sqrt{32}$ Take square roots of each side.

$x = 6 \pm \sqrt{32}$ Solve for x.

$x = 6 \pm 4\sqrt{2}$ Simplify: $\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

REVIEW RADICALS
For help with simplifying square roots, see p. 266.

▶ The solutions are $6 + 4\sqrt{2}$ and $6 - 4\sqrt{2}$.

CHECK You can use algebra or a graph.

Algebra Substitute each solution in the original equation to verify that it is correct.

Graph Use a graphing calculator to graph $y = x^2 - 12x + 4$. The x-intercepts are about $0.34 \approx 6 - 4\sqrt{2}$ and $11.66 \approx 6 + 4\sqrt{2}$.