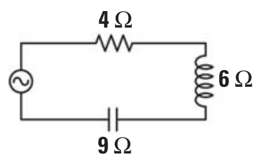
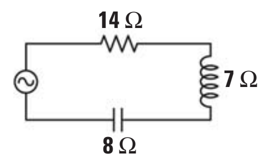
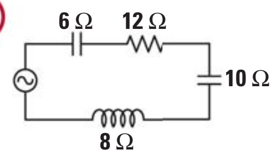
PROBLEM SOLVING

EXAMPLE 3 on p. 277 for Exs. 65–67

CIRCUITS In Exercises 65–67, each component of the circuit has been labeled with its resistance or reactance. Find the impedance of the circuit.

65. 

66. 

67. 

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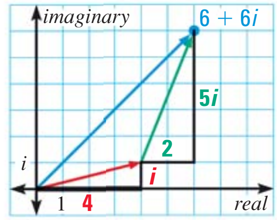
68. VISUAL THINKING The graph shows how you can geometrically add two complex numbers (in this case, $4 + i$ and $2 + 5i$) to find their sum (in this case, $6 + 6i$). Find each of the following sums by drawing a graph.



a. $(5 + i) + (1 + 4i)$ **b.** $(-7 + 3i) + (2 - 2i)$

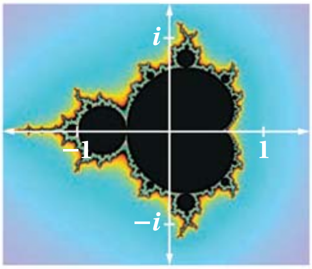
c. $(3 - 2i) + (-1 - i)$ **d.** $(4 + 2i) + (-5 - 3i)$

69. TAKS REASONING Make a table that shows the powers of i from i^1 to i^8 in the first row and the simplified forms of these powers in the second row. *Describe* the pattern you observe in the table. Verify that the pattern continues by evaluating the next four powers of i.

In Exercises 70–73, use the example below to determine whether the complex number c belongs to the Mandelbrot set. *Justify* your answer.

EXAMPLE **Investigate the Mandelbrot set**

Consider the function $f(z) = z^2 + c$ and this infinite list of complex numbers: $z_0 = 0$, $z_1 = f(z_0)$, $z_2 = f(z_1)$, $z_3 = f(z_2)$, ... If the absolute values of $z_0, z_1, z_2, z_3, \ldots$ are all less than some fixed number N, then c belongs to the *Mandelbrot set*. If the absolute values become infinitely large, then c does not belong to the Mandelbrot set.



The Mandelbrot set is the black region in the complex plane above.

Tell whether $c = 1 + i$ belongs to the Mandelbrot set.

Solution

Let $f(z) = z^2 + (1 + i)$.

$z_0 = \mathbf{0}$ $\qquad |z_0| = 0$

$z_1 = f(0) = 0^2 + (1 + i) = 1 + i$ $\qquad |z_1| \approx 1.41$

$z_2 = f(1 + i) = (1 + i)^2 + (1 + i) = 1 + 3i$ $\qquad |z_2| \approx 3.16$

$z_3 = f(1 + 3i) = (1 + 3i)^2 + (1 + i) = -7 + 7i$ $\qquad |z_3| \approx 9.90$

$z_4 = f(-7 + 7i) = (-7 + 7i)^2 + (1 + i) = 1 - 97i$ $\qquad |z_4| \approx 97.0$

▶ Because the absolute values are becoming infinitely large, $c = 1 + i$ does not belong to the Mandelbrot set.

70. $c = i$ **71.** $c = -1 + i$ **72.** $c = -1$ **73.** $c = -0.5i$