4.6 Perform Operations with Complex Numbers

TEKS 2A.2.B, 2A.8.A, 2A.8.D

Before You performed operations with real numbers.

Now You will perform operations with complex numbers.

Why? So you can solve problems involving fractals, as in Exs. 70–73.

Key Vocabulary
- **imaginary unit *i***
- **complex number**
- **imaginary number**
- **complex conjugates**
- **complex plane**
- **absolute value of a complex number**

Not all quadratic equations have real-number solutions. For example, $x^2 = -1$ has no real-number solutions because the square of any real number x is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the **imaginary unit *i***, defined as $i = \sqrt{-1}$. Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of *any* negative number.

KEY CONCEPT — *For Your Notebook*

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By Property (1), it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

EXAMPLE 1 Solve a quadratic equation

Solve $2x^2 + 11 = -37$.

$2x^2 + 11 = -37$	**Write original equation.**
$2x^2 = -48$	**Subtract 11 from each side.**
$x^2 = -24$	**Divide each side by 2.**
$x = \pm\sqrt{-24}$	**Take square roots of each side.**
$x = \pm i\sqrt{24}$	**Write in terms of *i*.**
$x = \pm 2i\sqrt{6}$	**Simplify radical.**

▶ The solutions are $2i\sqrt{6}$ and $-2i\sqrt{6}$.

✓ GUIDED PRACTICE for Example 1

Solve the equation.

1. $x^2 = -13$

2. $x^2 = -38$

3. $x^2 + 11 = 3$

4. $x^2 - 8 = -36$

5. $3x^2 - 7 = -31$

6. $5x^2 + 33 = 3$