

### RATIONALIZING THE DENOMINATOR

Suppose the denominator of a fraction has the form  $\sqrt{b}$ ,  $a + \sqrt{b}$ , or  $a - \sqrt{b}$  where  $a$  and  $b$  are rational numbers. The table shows how to eliminate the radical from the denominator. This is called **rationalizing the denominator**.

Form of the denominator	Multiply numerator and denominator by:
$\sqrt{b}$	$\sqrt{b}$
$a + \sqrt{b}$	$a - \sqrt{b}$
$a - \sqrt{b}$	$a + \sqrt{b}$

The expressions  $a + \sqrt{b}$  and  $a - \sqrt{b}$  are called **conjugates** of each other. Their product is always a rational number.

### EXAMPLE 2 Rationalize denominators of fractions

Simplify (a)  $\sqrt{\frac{5}{2}}$  and (b)  $\frac{3}{7 + \sqrt{2}}$ .

**Solution**

$$\begin{aligned} \text{a. } \sqrt{\frac{5}{2}} &= \frac{\sqrt{5}}{\sqrt{2}} \\ &= \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{10}}{2} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{3}{7 + \sqrt{2}} &= \frac{3}{7 + \sqrt{2}} \cdot \frac{7 - \sqrt{2}}{7 - \sqrt{2}} \\ &= \frac{21 - 3\sqrt{2}}{49 - 7\sqrt{2} + 7\sqrt{2} - 2} \\ &= \frac{21 - 3\sqrt{2}}{47} \end{aligned}$$

**SOLVING QUADRATIC EQUATIONS** You can use square roots to solve some types of quadratic equations. For example, if  $s > 0$ , then the equation  $x^2 = s$  has two real-number solutions:  $x = \sqrt{s}$  and  $x = -\sqrt{s}$ . These solutions are often written in condensed form as  $x = \pm\sqrt{s}$  (read as “plus or minus the square root of  $s$ ”).

### EXAMPLE 3 Solve a quadratic equation

Solve  $3x^2 + 5 = 41$ .

$$3x^2 + 5 = 41$$

**Write original equation.**

$$3x^2 = 36$$

**Subtract 5 from each side.**

$$x^2 = 12$$

**Divide each side by 3.**

$$x = \pm\sqrt{12}$$

**Take square roots of each side.**

$$x = \pm\sqrt{4} \cdot \sqrt{3}$$

**Product property**

$$x = \pm 2\sqrt{3}$$

**Simplify.**

► The solutions are  $2\sqrt{3}$  and  $-2\sqrt{3}$ .

**CHECK** Check the solutions by substituting them into the original equation.

$$3x^2 + 5 = 41$$

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$$3(2\sqrt{3})^2 + 5 \stackrel{?}{=} 41$$

$$3(-2\sqrt{3})^2 + 5 \stackrel{?}{=} 41$$

$$3(12) + 5 \stackrel{?}{=} 41$$

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$$41 = 41 \checkmark$$

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#### AVOID ERRORS

When solving an equation of the form  $x^2 = s$  where  $s > 0$ , make sure to find both the positive and negative solutions.