FACTORING AND ZEROS To find the maximum or minimum value of a quadratic function, you can first use factoring to write the function in intercept form $y = a(x - p)(x - q)$. Because the function's vertex lies on the axis of symmetry $x = \frac{p+q}{2}$, the maximum or minimum occurs at the *average* of the zeros p and q.

TAKS

EXAMPLE 7 TAKS REASONING: Multi-Step Problem

MAGAZINES A monthly teen magazine has 28,000 subscribers when it charges \$10 per annual subscription. For each \$1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

Solution

STEP 1 **Define** the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

STEP 2 **Write** a verbal model. Then write and simplify a quadratic function.

Annual revenue (dollars)	=	Number of subscribers (people)	•	Subscription price (dollars/person)
$R(x)$	=	$(28{,}000 - 2000x)$	•	$(10 + x)$

$$R(x) = (-2000x + 28{,}000)(x + 10)$$

$$R(x) = -2000(x - 14)(x + 10)$$

STEP 3 **Identify** the zeros and find their average. Find how much each subscription should cost to maximize annual revenue.

The zeros of the revenue function are 14 and -10. The average of the zeros is $\frac{14 + (-10)}{2} = 2$. To maximize revenue, each subscription should cost $\$10 + \$2 = \$12$.

STEP 4 **Find** the maximum annual revenue.

$R(2) = -2000(2 - 14)(2 + 10) = \$288{,}000$

▶ The magazine should charge \$12 per subscription to maximize annual revenue. The maximum annual revenue is \$288,000.

✓ GUIDED PRACTICE for Examples 5, 6, and 7

Solve the equation.

19. $6x^2 - 3x - 63 = 0$ **20.** $12x^2 + 7x + 2 = x + 8$ **21.** $7x^2 + 70x + 175 = 0$

22. **WHAT IF?** In Example 7, suppose the magazine initially charges \$11 per annual subscription. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?