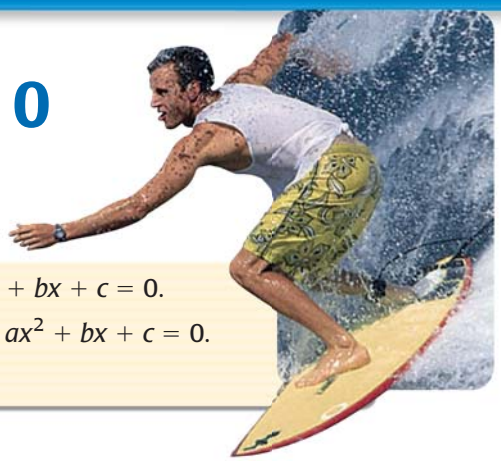
4.4 Solve $ax^2 + bx + c = 0$ by Factoring



TEKS 2A.2.A, 2A.6.B, 2A.8.A, 2A.8.D

Before You used factoring to solve equations of the form $x^2 + bx + c = 0$.

Now You will use factoring to solve equations of the form $ax^2 + bx + c = 0$.

Why? So you can maximize a shop's revenue, as in Ex. 64.

Key Vocabulary
- **monomial,** *p. 252*

To factor $ax^2 + bx + c$ when $a \neq 1$, find integers k, l, m, and n such that:

$$ax^2 + bx + c = (kx + m)(lx + n) = klx^2 + (kn + lm)x + mn$$

So, k and l must be factors of a, and m and n must be factors of c.

EXAMPLE 1 Factor $ax^2 + bx + c$ where $c > 0$

Factor $5x^2 - 17x + 6$.

Solution

You want $5x^2 - 17x + 6 = (kx + m)(lx + n)$ where k and l are factors of 5 and m and n are factors of 6. You can assume that k and l are positive and $k \geq l$. Because $mn > 0$, m and n have the same sign. So, m and n must both be negative because the coefficient of x, -17, is negative.

FACTOR EXPRESSIONS
When factoring $ax^2 + bx + c$ where $a > 0$, it is customary to choose factors $kx + m$ and $lx + n$ such that k and l are positive.

k, l	5, 1	5, 1	5, 1	**5, 1**
m, n	$-6, -1$	$-1, -6$	$-3, -2$	**$-2, -3$**
$(kx + m)(lx + n)$	$(5x - 6)(x - 1)$	$(5x - 1)(x - 6)$	$(5x - 3)(x - 2)$	**$(5x - 2)(x - 3)$**
$ax^2 + bx + c$	$5x^2 - 11x + 6$	$5x^2 - 31x + 6$	$5x^2 - 13x + 6$	**$5x^2 - 17x + 6$**

▶ The correct factorization is $5x^2 - 17x + 6 = (5x - 2)(x - 3)$.

EXAMPLE 2 Factor $ax^2 + bx + c$ where $c < 0$

Factor $3x^2 + 20x - 7$.

Solution

You want $3x^2 + 20x - 7 = (kx + m)(lx + n)$ where k and l are factors of 3 and m and n are factors of -7. Because $mn < 0$, m and n have opposite signs.

k, l	3, 1	**3, 1**	3, 1	3, 1
m, n	$7, -1$	**$-1, 7$**	$-7, 1$	$1, -7$
$(kx + m)(lx + n)$	$(3x + 7)(x - 1)$	**$(3x - 1)(x + 7)$**	$(3x - 7)(x + 1)$	$(3x + 1)(x - 7)$
$ax^2 + bx + c$	$3x^2 + 4x - 7$	**$3x^2 + 20x - 7$**	$3x^2 - 4x - 7$	$3x^2 - 20x - 7$

▶ The correct factorization is $3x^2 + 20x - 7 = (3x - 1)(x + 7)$.