4.3 Solve $x^2 + bx + c = 0$ by Factoring

TEKS 2A.2.A, 2A.6.A, 2A.8.A, 2A.8.D

Before You graphed quadratic functions.

Now You will solve quadratic equations.

Why? So you can double the area of a picnic site, as in Ex. 42.

Key Vocabulary
- monomial
- binomial
- trinomial
- quadratic equation
- root of an equation
- zero of a function

A **monomial** is an expression that is either a number, a variable, or the product of a number and one or more variables. A **binomial**, such as $x + 4$, is the sum of two monomials. A **trinomial**, such as $x^2 + 11x + 28$, is the sum of three monomials.

You know how to use FOIL to write $(x + 4)(x + 7)$ as $x^2 + 11x + 28$. You can use factoring to write a trinomial as a product of binomials. To factor $x^2 + bx + c$, find integers m and n such that:

$$x^2 + bx + c = (x + m)(x + n)$$
$$= x^2 + (m + n)x + mn$$

So, the *sum* of m and n must equal b and the *product* of m and n must equal c.

EXAMPLE 1 Factor trinomials of the form $x^2 + bx + c$

Factor the expression.

a. $x^2 - 9x + 20$ **b.** $x^2 + 3x - 12$

Solution

a. You want $x^2 - 9x + 20 = (x + m)(x + n)$ where $mn = 20$ and $m + n = -9$.

AVOID ERRORS
When factoring $x^2 + bx + c$ where $c > 0$, you must choose factors $x + m$ and $x + n$ such that m and n have the same sign.

Factors of 20: m, n	1, 20	−1, −20	2, 10	−2, −10	4, 5	**−4, −5**
Sum of factors: $m + n$	21	−21	12	−12	9	**−9**

▶ Notice that $m = -4$ and $n = -5$. So, $x^2 - 9x + 20 = (x - 4)(x - 5)$.

b. You want $x^2 + 3x - 12 = (x + m)(x + n)$ where $mn = -12$ and $m + n = 3$.

Factors of −12: m, n	−1, 12	1, −12	−2, 6	2, −6	−3, 4	3, −4
Sum of factors: $m + n$	11	−11	4	−4	1	−1

▶ Notice that there are no factors m and n such that $m + n = 3$. So, $x^2 + 3x - 12$ cannot be factored.

✓ GUIDED PRACTICE for Example 1

Factor the expression. If the expression cannot be factored, say so.

1. $x^2 - 3x - 18$ **2.** $n^2 - 3n + 9$ **3.** $r^2 + 2r - 63$