3.8 Use Inverse Matrices to Solve Linear Systems

TEKS 2A.2.A, 2A.3.A, 2A.3.B, 2A.3.C

Before You solved linear systems using Cramer's rule.

Now You will solve linear systems using inverse matrices.

Why? So you can find how many batches of a recipe to make, as in Ex. 45.

Key Vocabulary
- identity matrix
- inverse matrices
- matrix of variables
- matrix of constants

The $n \times n$ **identity matrix** is a matrix with 1's on the main diagonal and 0's elsewhere. If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $IA = A$.

2 × 2 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two $n \times n$ matrices A and B are **inverses** of each other if their product (in both orders) is the $n \times n$ identity matrix. That is, $AB = I$ and $BA = I$. An $n \times n$ matrix A has an inverse if and only if det $A \neq 0$. The symbol for the inverse of A is A^{-1}.

KEY CONCEPT — *For Your Notebook*

The Inverse of a 2 × 2 Matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ provided $ad - cb \neq 0$.

EXAMPLE 1 Find the inverse of a 2 × 2 matrix

Find the inverse of $A = \begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix}$.

$$A^{-1} = \frac{1}{15 - 16}\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix} = -1\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -5 & 8 \\ 2 & -3 \end{bmatrix}$$

CHECK INVERSES
In Example 1, you can check the inverse by showing that $AA^{-1} = I = A^{-1}A$.

GUIDED PRACTICE for Example 1

Find the inverse of the matrix.

1. $\begin{bmatrix} 6 & 1 \\ 2 & 4 \end{bmatrix}$ **2.** $\begin{bmatrix} -1 & 5 \\ -4 & 8 \end{bmatrix}$ **3.** $\begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$