KEY CONCEPT *For Your Notebook*

Cramer's Rule for a 3 × 3 System

Let A be the coefficient matrix of the linear system shown below.

Linear System

$$\begin{aligned} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{aligned}$$

Coefficient Matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}, \quad \text{and} \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}$$

SOLVE SYSTEMS

As with Cramer's rule for a 2 × 2 system, the numerators for *x*, *y*, and *z* are the determinants of the matrices formed by replacing the coefficients of *x*, *y*, and *z* respectively with the column of constants.

EXAMPLE 4 TAKS REASONING: Multi-Step Problem

CHEMISTRY The atomic weights of three compounds are shown. Use a linear system and Cramer's rule to find the atomic weights of carbon (C), hydrogen (H), and oxygen (O).

Compound	Formula	Atomic weight
Glucose	$C_6H_{12}O_6$	180
Carbon dioxide	CO_2	44
Hydrogen peroxide	H_2O_2	34

Solution

STEP 1 **Write** a linear system using the formula for each compound. Let C, H, and O represent the atomic weights of carbon, hydrogen, and oxygen.

$$\begin{aligned} 6C + 12H + 6O &= 180 \\ C \quad\quad + 2O &= 44 \\ 2H + 2O &= 34 \end{aligned}$$

STEP 2 **Evaluate** the determinant of the coefficient matrix.

$$\begin{vmatrix} 6 & 12 & 6 \\ 1 & 0 & 2 \\ 0 & 2 & 2 \end{vmatrix} \begin{matrix} 6 & 12 \\ 1 & 0 \\ 0 & 2 \end{matrix} = (0 + 0 + 12) - (0 + 24 + 24) = -36$$

STEP 3 **Apply** Cramer's rule because the determinant is not 0.

$$C = \frac{\begin{vmatrix} 180 & 12 & 6 \\ 44 & 0 & 2 \\ 34 & 2 & 2 \end{vmatrix}}{-36} \qquad H = \frac{\begin{vmatrix} 6 & 180 & 6 \\ 1 & 44 & 2 \\ 0 & 34 & 2 \end{vmatrix}}{-36} \qquad O = \frac{\begin{vmatrix} 6 & 12 & 180 \\ 1 & 0 & 44 \\ 0 & 2 & 34 \end{vmatrix}}{-36}$$

$$= \frac{-432}{-36} \qquad = \frac{-36}{-36} \qquad = \frac{-576}{-36}$$

$$= 12 \qquad = 1 \qquad = 16$$

▶ The atomic weights of carbon, hydrogen, and oxygen are 12, 1, and 16, respectively.