

CRAMER'S RULE You can use determinants to solve a system of linear equations. The method, called **Cramer's rule** and named after the Swiss mathematician Gabriel Cramer (1704–1752), uses the **coefficient matrix** of the linear system.

Linear System	Coefficient Matrix
$ax + by = e$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
$cx + dy = f$	

KEY CONCEPT

For Your Notebook

Cramer's Rule for a 2×2 System

Let A be the coefficient matrix of this linear system:

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Notice that the numerators for x and y are the determinants of the matrices formed by replacing the coefficients of x and y , respectively, with the column of constants.

EXAMPLE 3 Use Cramer's rule for a 2×2 system

Use Cramer's rule to solve this system: $\begin{aligned} 9x + 4y &= -6 \\ 3x - 5y &= -21 \end{aligned}$

Solution

STEP 1 Evaluate the determinant of the coefficient matrix.

$$\begin{vmatrix} 9 & 4 \\ 3 & -5 \end{vmatrix} = -45 - 12 = -57$$

STEP 2 Apply Cramer's rule because the determinant is not 0.

$$x = \frac{\begin{vmatrix} -6 & 4 \\ -21 & -5 \end{vmatrix}}{-57} = \frac{30 - (-84)}{-57} = \frac{114}{-57} = -2$$

$$y = \frac{\begin{vmatrix} 9 & -6 \\ 3 & -21 \end{vmatrix}}{-57} = \frac{-189 - (-18)}{-57} = \frac{-171}{-57} = 3$$

► The solution is $(-2, 3)$.

CHECK Check this solution in the original equations.

$9x + 4y = -6$	$3x - 5y = -21$
$9(-2) + 4(3) \stackrel{?}{=} -6$	$3(-2) - 5(3) \stackrel{?}{=} -21$
$-18 + 12 \stackrel{?}{=} -6$	$-6 - 15 \stackrel{?}{=} -21$
$-6 = -6 \checkmark$	$-21 = -21 \checkmark$

ANOTHER WAY

You can also solve the system in Example 3 using the substitution method or the elimination method you learned in Lesson 3.2.