

AREA OF A TRIANGLE You can use a determinant to find the area of a triangle whose vertices are points in a coordinate plane.

KEY CONCEPT

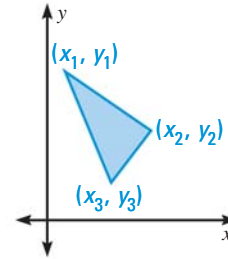
For Your Notebook

Area of a Triangle

The area of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is given by

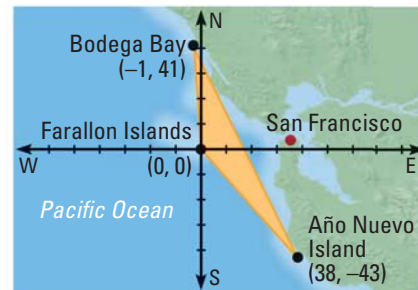
$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol \pm indicates that the appropriate sign should be chosen to yield a positive value.



EXAMPLE 2 Find the area of a triangular region

SEA LIONS Off the coast of California lies a triangular region of the Pacific Ocean where huge populations of sea lions and seals live. The triangle is formed by imaginary lines connecting Bodega Bay, the Farallon Islands, and Año Nuevo Island, as shown. (In the map, the coordinates are measured in miles.) Use a determinant to estimate the area of the region.



Solution

The approximate coordinates of the vertices of the triangular region are $(-1, 41)$, $(38, -43)$, and $(0, 0)$. So, the area of the region is:

$$\begin{aligned} \text{Area} &= \pm \frac{1}{2} \begin{vmatrix} -1 & 41 & 1 \\ 38 & -43 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm \frac{1}{2} \begin{vmatrix} -1 & 41 & 1 \\ 38 & -43 & 1 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} -1 & 41 \\ 38 & -43 \end{vmatrix} \\ &= \pm \frac{1}{2} [(43 + 0 + 0) - (0 + 0 + 1558)] \\ &= 757.5 \end{aligned}$$

► The area of the region is about 758 square miles.



GUIDED PRACTICE for Examples 1 and 2

Evaluate the determinant of the matrix.

1. $\begin{bmatrix} 3 & -2 \\ 6 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 4 & -1 & 2 \\ -3 & -2 & -1 \\ 0 & 5 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 10 & -2 & 3 \\ 2 & -12 & 4 \\ 0 & -7 & -2 \end{bmatrix}$

4. Find the area of the triangle with vertices $A(5, 11)$, $B(9, 2)$, and $C(1, 3)$.