3.7 Evaluate Determinants and Apply Cramer's Rule

TEKS a.1, a.4, 2A.3.A, 2A.3.B

Before You added, subtracted, and multiplied matrices.

Now You will evaluate determinants of matrices.

Why? So you can find areas of habitats, as in Example 2.

Key Vocabulary
- **determinant**
- **Cramer's rule**
- **coefficient matrix**

Associated with each square $(n \times n)$ matrix is a real number called its **determinant**. The determinant of a matrix A is denoted by $\det A$ or by $|A|$.

KEY CONCEPT *For Your Notebook*

The Determinant of a Matrix

Determinant of a 2 × 2 Matrix

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

The determinant of a 2 × 2 matrix is the difference of the products of the elements on the diagonals.

Determinant of a 3 × 3 Matrix

STEP 1 **Repeat** the first two columns to the right of the determinant.

STEP 2 **Subtract** the sum of the **red products** from the sum of the **blue products**.

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

EXAMPLE 1 Evaluate determinants

Evaluate the determinant of the matrix.

a. $\begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & -1 & -3 \\ 4 & 1 & 0 \\ 3 & -4 & -2 \end{bmatrix}$

Solution

a. $\begin{vmatrix} 5 & 4 \\ 3 & 1 \end{vmatrix} = 5(1) - 3(4) = 5 - 12 = -7$

b. $\begin{vmatrix} 2 & -1 & -3 & 2 & -1 \\ 4 & 1 & 0 & 4 & 1 \\ 3 & -4 & -2 & 3 & -4 \end{vmatrix} = (-4 + 0 + 48) - (-9 + 0 + 8) = 44 - (-1) = 45$