

EXAMPLE 4 Solve a system using substitution

MARKETING The marketing department of a company has a budget of \$30,000 for advertising. A television ad costs \$1000, a radio ad costs \$200, and a newspaper ad costs \$500. The department wants to run 60 ads per month and have as many radio ads as television and newspaper ads combined. How many of each type of ad should the department run each month?

Solution

STEP 1 Write verbal models for the situation.

$$\begin{array}{|c|} \hline \text{TV} \\ \hline \text{ads} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{Radio} \\ \hline \text{ads} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{Newspaper} \\ \hline \text{ads} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Total} \\ \hline \text{ads} \\ \hline \end{array} \quad \text{Equation 1}$$

$$1000 \cdot \begin{array}{|c|} \hline \text{TV} \\ \hline \text{ads} \\ \hline \end{array} + 200 \cdot \begin{array}{|c|} \hline \text{Radio} \\ \hline \text{ads} \\ \hline \end{array} + 500 \cdot \begin{array}{|c|} \hline \text{Newspaper} \\ \hline \text{ads} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Monthly} \\ \hline \text{budget} \\ \hline \end{array} \quad \text{Equation 2}$$

$$\begin{array}{|c|} \hline \text{Radio} \\ \hline \text{ads} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{TV} \\ \hline \text{ads} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{Newspaper} \\ \hline \text{ads} \\ \hline \end{array} \quad \text{Equation 3}$$

STEP 2 Write a system of equations. Let x be the number of TV ads, y be the number of radio ads, and z be the number of newspaper ads.

$$\begin{array}{rcl} x + y + z = 60 & \text{Equation 1} \\ 1000x + 200y + 500z = 30,000 & \text{Equation 2} \\ y = x + z & \text{Equation 3} \end{array}$$

STEP 3 Rewrite the system in Step 2 as a linear system in *two* variables by substituting $x + z$ for y in Equations 1 and 2.

$$\begin{array}{rcl} x + y + z = 60 & \text{Write Equation 1.} \\ x + (x + z) + z = 60 & \text{Substitute } x + z \text{ for } y. \\ 2x + 2z = 60 & \text{New Equation 1} \\ \\ 1000x + 200y + 500z = 30,000 & \text{Write Equation 2.} \\ 1000x + 200(x + z) + 500z = 30,000 & \text{Substitute } x + z \text{ for } y. \\ 1200x + 700z = 30,000 & \text{New Equation 2} \end{array}$$

STEP 4 Solve the linear system in two variables from Step 3.

$$\begin{array}{rcl} -1200x - 1200z = -36,000 & \text{Add } -600 \text{ times new Equation 1} \\ 1200x + 700z = 30,000 & \text{to new Equation 2.} \\ \hline -500z = -6000 & \\ z = 12 & \text{Solve for } z. \\ x = 18 & \text{Substitute into new Equation 1 or 2 to find } x. \\ y = 30 & \text{Substitute into an original equation to find } y. \end{array}$$

► The solution is $x = 18$, $y = 30$, and $z = 12$, or $(18, 30, 12)$. So, the department should run 18 TV ads, 30 radio ads, and 12 newspaper ads each month.

AVOID ERRORS

In Example 4, be careful not to write the ordered triple in the order in which you solved for the variables.

- $(12, 18, 30)$ ✗
 $(18, 30, 12)$ ✓

**GUIDED PRACTICE** for Example 4

4. **WHAT IF?** In Example 4, suppose the monthly budget is \$25,000. How many of each type of ad should the marketing department run each month?