ELIMINATION METHOD The elimination method you studied in Lesson 3.2 can be extended to solve a system of linear equations in three variables.

KEY CONCEPT *For Your Notebook*

The Elimination Method for a Three-Variable System

STEP 1 **Rewrite** the linear system in three variables as a linear system in two variables by using the elimination method.

STEP 2 **Solve** the new linear system for both of its variables.

STEP 3 **Substitute** the values found in Step 2 into one of the original equations and solve for the remaining variable.

If you obtain a false equation, such as $0 = 1$, in any of the steps, then the system has no solution.

If you do not obtain a false equation, but obtain an identity such as $0 = 0$, then the system has infinitely many solutions.

EXAMPLE 1 Use the elimination method

Solve the system.

$4x + 2y + 3z = 1$ **Equation 1**

$2x - 3y + 5z = -14$ **Equation 2**

$6x - y + 4z = -1$ **Equation 3**

Solution

ANOTHER WAY

In Step 1, you could also eliminate *x* to get two equations in *y* and *z*, or you could eliminate *z* to get two equations in *x* and *y*.

STEP 1 **Rewrite** the system as a linear system in *two* variables.

$$\begin{aligned} 4x + 2y + 3z &= 1 \\ 12x - 2y + 8z &= -2 \\ \hline 16x \qquad + 11z &= -1 \end{aligned}$$

Add 2 times Equation 3 to Equation 1. → **New Equation 1**

$$\begin{aligned} 2x - 3y + 5z &= -14 \\ -18x + 3y - 12z &= 3 \\ \hline -16x \qquad - 7z &= -11 \end{aligned}$$

Add −3 times Equation 3 to Equation 2. → **New Equation 2**

STEP 2 **Solve** the new linear system for both of its variables.

$$\begin{aligned} 16x + 11z &= -1 \\ -16x - 7z &= -11 \\ \hline 4z &= -12 \end{aligned}$$

Add new Equation 1 and new Equation 2.

$z = -3$ **Solve for *z*.**

$x = 2$ **Substitute into new Equation 1 or 2 to find *x*.**

STEP 3 **Substitute** $x = 2$ and $z = -3$ into an original equation and solve for *y*.

$6x - y + 4z = -1$ **Write original Equation 3.**

$6(2) - y + 4(-3) = -1$ **Substitute 2 for *x* and −3 for *z*.**

$y = 1$ **Solve for *y*.**

▶ The solution is $x = 2$, $y = 1$, and $z = -3$, or the ordered triple $(2, 1, -3)$. Check this solution in each of the original equations.