3.4 Solve Systems of Linear Equations in Three Variables

TEKS a.5, 2A.3.A, 2A.3.B, 2A.3.C

Before You solved systems of equations in two variables.

Now You will solve systems of equations in three variables.

Why? So you can model the results of a sporting event, as in Ex. 45.

Key Vocabulary
- **linear equation in three variables**
- **system of three linear equations**
- **solution of a system of three linear equations**
- **ordered triple**

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$ where a, b, and c are not all zero.

The following is an example of a **system of three linear equations** in three variables.

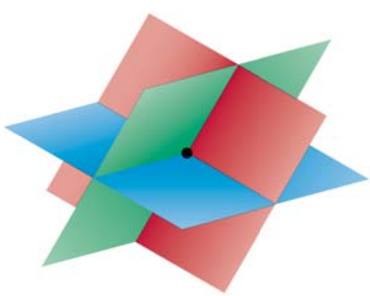
$$2x + y - z = 5 \quad \text{Equation 1}$$
$$3x - 2y + z = 16 \quad \text{Equation 2}$$
$$4x + 3y - 5z = 3 \quad \text{Equation 3}$$

A **solution** of such a system is an **ordered triple** (x, y, z) whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in three-dimensional space. The graphs of three such equations that form a system are three planes whose intersection determines the number of solutions of the system, as shown in the diagrams below.

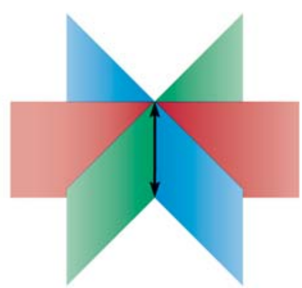
Exactly one solution

The planes intersect in a single point.



Infinitely many solutions

The planes intersect in a line or are the same plane.



No solution

The planes have no common point of intersection.