3.4 Graphing Linear Equations in Three Variables

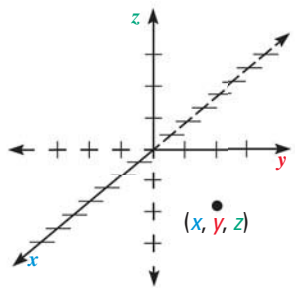
TEKS a.4, a.5

MATERIALS • graph paper • ruler

QUESTION What is the graph of a linear equation in three variables?

A *linear equation in three variables* has the form $ax + by + cz = d$. You can graph this type of equation in a three-dimensional coordinate system formed by three axes that divide space into eight *octants*. Each point in space is represented by an *ordered triple* (x, y, z).

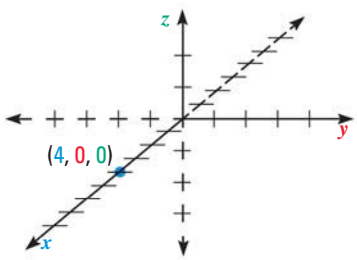
The graph of any equation in three variables is the set of all points (x, y, z) whose coordinates make the equation true. For a linear equation in three variables, the graph is a plane.



EXPLORE Graph 3x + 4y + 6z = 12

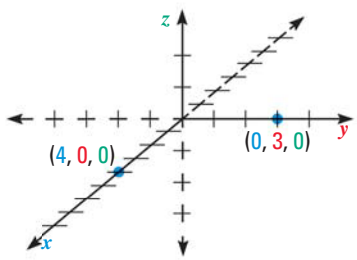
STEP 1 Find x-intercept

Find the x-intercept by setting y and z equal to 0 and solving the resulting equation, $3x = 12$. The x-intercept is 4, so plot (4, 0, 0).



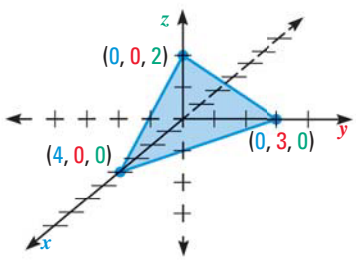
STEP 2 Find y-intercept

Find the y-intercept by setting x and z equal to 0 and solving the resulting equation, $4y = 12$. The y-intercept is 3, so plot (0, 3, 0).



STEP 3 Find z-intercept

Find the z-intercept by setting x and y equal to 0 and solving the resulting equation, $6z = 12$. The z-intercept is 2, so plot (0, 0, 2). Then connect the points.



The triangular region shown in Step 3 is the portion of the graph of $3x + 4y + 6z = 12$ that lies in the first octant.

DRAW CONCLUSIONS Use your observations to complete these exercises

Sketch the graph of the equation.

1. $4x + 3y + 2z = 12$
2. $2x + 2y + 3z = 6$
3. $x + 5y + 3z = 15$
4. $5x - y - 2z = 10$
5. $-7x + 7y + 2z = 14$
6. $2x + 9y - 3z = -18$

7. Suppose three linear equations in three variables are graphed in the same coordinate system. In how many different ways can the planes intersect? *Explain* your reasoning.