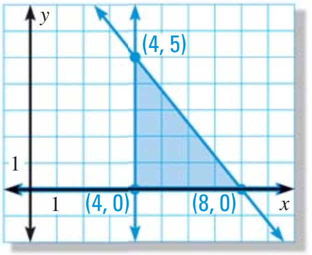
EXAMPLE 1 Use linear programming to maximize profit

BUSINESS How many bowls and how many plates should the potter described on page 174 make in order to maximize profit?

Solution

STEP 1 **Graph** the system of constraints:

$x \geq 4$	**Make at least 4 bowls.**
$y \geq 0$	**Number of plates cannot be negative.**
$5x + 4y \leq 40$	**Can use up to 40 pounds of clay.**



STEP 2 **Evaluate** the profit function $P = 35x + 30y$ at each vertex of the feasible region.

At (4, 0): $P = 35(4) + 30(0) = 140$
At (8, 0): $P = 35(8) + 30(0) = 280$
At (4, 5): $P = 35(4) + 30(5) = 290$ ← **Maximum**

▶ The potter can maximize profit by making 4 bowls and 5 plates.

EXAMPLE 2 Solve a linear programming problem

Find the minimum value and the maximum value of the objective function $C = 4x + 5y$ subject to the following constraints.

$x \geq 0$

$y \geq 0$

$x + 2y \leq 16$

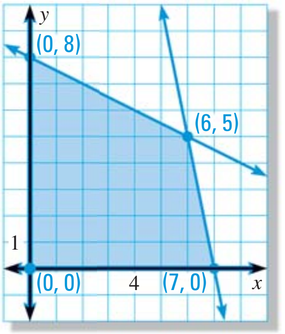
$5x + y \leq 35$

Solution

STEP 1 **Graph** the system of constraints. Find the coordinates of the vertices of the feasible region by solving systems of two linear equations. For example, the solution of the system

$$x + 2y = 16$$
$$5x + y = 35$$

gives the vertex (6, 5). The other three vertices are (0, 0), (7, 0), and (0, 8).



STEP 2 **Evaluate** the function $C = 4x + 5y$ at each of the vertices.

At (0, 0): $C = 4(0) + 5(0) = 0$ ← **Minimum**
At (7, 0): $C = 4(7) + 5(0) = 28$
At (6, 5): $C = 4(6) + 5(5) = 49$ ← **Maximum**
At (0, 8): $C = 4(0) + 5(8) = 40$

▶ The minimum value of C is 0. It occurs when $x = 0$ and $y = 0$.
The maximum value of C is 49. It occurs when $x = 6$ and $y = 5$.