TRANSFORMATIONS OF ANY GRAPH You can perform transformations on the graph of *any* function f in the same way as for absolute value graphs.

KEY CONCEPT *For Your Notebook*

Transformations of General Graphs

The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of any function $y = f(x)$ by performing these steps:

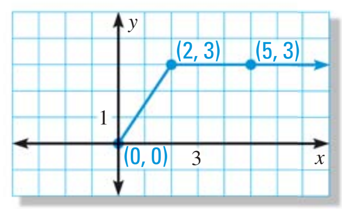
STEP 1 **Stretch or shrink** the graph of $y = f(x)$ vertically by a factor of $|a|$ if $|a| \neq 1$. If $|a| > 1$, stretch the graph. If $|a| < 1$, shrink the graph.

STEP 2 **Reflect** the resulting graph from Step 1 in the x-axis if $a < 0$.

STEP 3 **Translate** the resulting graph from Step 2 horizontally h units and vertically k units.

EXAMPLE 5 Apply transformations to a graph

The graph of a function $y = f(x)$ is shown. Sketch the graph of the given function.

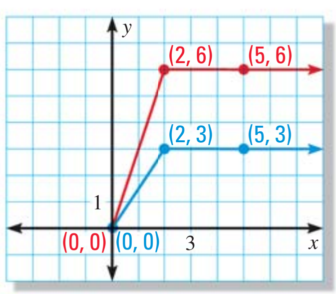


a. $y = 2 \cdot f(x)$

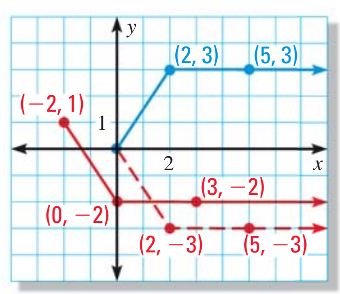
b. $y = -f(x + 2) + 1$

Solution

a. The graph of $y = 2 \cdot f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. (There is no reflection or translation.) To draw the graph, multiply the y-coordinate of each labeled point on the graph of $y = f(x)$ by 2 and connect their images.



b. The graph of $y = -f(x + 2) + 1$ is the graph of $y = f(x)$ reflected in the x-axis, then translated left 2 units and up 1 unit. To draw the graph, first reflect the labeled points and connect their images. Then translate and connect these points to form the final image.



AVOID ERRORS
In Example 5, part (b), the value of h is -2 because $-f(x + 2) + 1 = -f(x - (-2)) + 1$. Because $-2 < 0$, the horizontal translation is to the left.

✓ **GUIDED PRACTICE** for Examples 4 and 5

4. WHAT IF? In Example 4, suppose the reference beam originates at (3, 0) and reflects off a mirror at (5, 4). Write an equation for the path of the beam.

Use the graph of $y = f(x)$ from Example 5 to graph the given function.

5. $y = 0.5 \cdot f(x)$

6. $y = -f(x - 2) - 5$

7. $y = 2 \cdot f(x + 3) - 1$