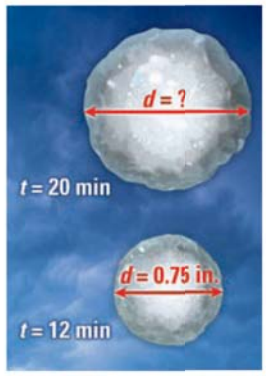
EXAMPLE 2 Write and apply a model for direct variation

METEOROLOGY Hailstones form when strong updrafts support ice particles high in clouds, where water droplets freeze onto the particles. The diagram shows a hailstone at two different times during its formation.



a. Write an equation that gives the hailstone's diameter d (in inches) after t minutes if you assume the diameter varies directly with the time the hailstone takes to form.

b. Using your equation from part (a), predict the diameter of the hailstone after 20 minutes.

Solution

a. Use the given values of t and d to find the constant of variation.

$d = at$ Write direct variation equation.

$0.75 = a(12)$ Substitute 0.75 for d and 12 for t.

$0.0625 = a$ Solve for a.

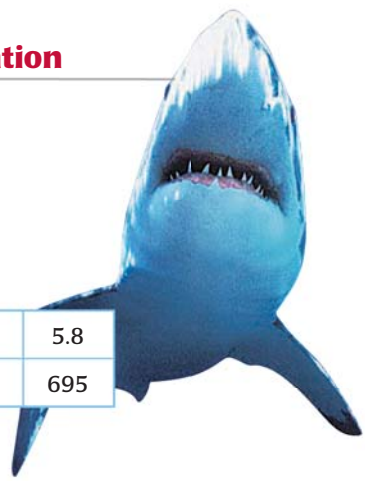
An equation that relates t and d is $d = 0.0625t$.

b. After $t = 20$ minutes, the predicted diameter of the hailstone is $d = 0.0625(20) = 1.25$ inches.

RATIOS AND DIRECT VARIATION Because the direct variation equation $y = ax$ can be written as $\frac{y}{x} = a$, a set of data pairs (x, y) shows direct variation if the ratio of y to x is constant.

EXAMPLE 3 Use ratios to identify direct variation

SHARKS Great white sharks have triangular teeth. The table below gives the length of a side of a tooth and the body length for each of six great white sharks. Tell whether tooth length and body length show direct variation. If so, write an equation that relates the quantities.



Tooth length, t (cm)	1.8	2.4	2.9	3.6	4.7	5.8
Body length, b (cm)	215	290	350	430	565	695

Solution

Find the ratio of the body length b to the tooth length t for each shark.

$\frac{215}{1.8} \approx 119$ $\quad \frac{290}{2.4} \approx 121$ $\quad \frac{350}{2.9} = 121$

$\frac{430}{3.6} \approx 119$ $\quad \frac{565}{4.7} \approx 120$ $\quad \frac{695}{5.8} \approx 120$

Because the ratios are approximately equal, the data show direct variation.

An equation relating tooth length and body length is $\frac{b}{t} = 120$, or $b = 120t$.

AVOID ERRORS
For real-world data, the ratios do not have to be *exactly* the same to show that direct variation is a plausible model.