

## Properties of Radicals and Rational Exponents

### Number of Real $n$ th Roots (p. 414)

Let  $n$  be an integer greater than 1, and let  $a$  be a real number.

- If  $n$  is odd, then  $a$  has one real  $n$ th root:  $\sqrt[n]{a} = a^{1/n}$
- If  $n$  is even and  $a > 0$ , then  $a$  has two real  $n$ th roots:  $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If  $n$  is even and  $a = 0$ , then  $a$  has one  $n$ th root:  $\sqrt[n]{0} = 0^{1/n} = 0$
- If  $n$  is even and  $a < 0$ , then  $a$  has no real  $n$ th roots.

### Radicals and Rational Exponents (p. 415)

Let  $a^{1/n}$  be an  $n$ th root of  $a$ , and let  $m$  be a positive integer.

- $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$
- $a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m}, a \neq 0$

### Properties of Rational Exponents (p. 420)

All of the properties of exponents listed on the previous page apply to rational exponents as well as integer exponents.

### Product and Quotient Properties of Radicals (p. 421)

Let  $n$  be an integer greater than 1, and let  $a$  and  $b$  be positive real numbers. Then  $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$  and  $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ .

## Properties of Logarithms

### Logarithms and Exponents (p. 499)

Let  $a, b, c, m, n, x$ , and  $y$  be positive real numbers such that  $b \neq 1$  and  $c \neq 1$ .

$$\log_b y = x \text{ if and only if } b^x = y$$

### Special Logarithm Values (p. 499)

$$\log_b 1 = 0 \text{ because } b^0 = 1 \text{ and } \log_b b = 1 \text{ because } b^1 = b$$

### Common and Natural Logarithms (p. 500)

$$\log_{10} x = \log x \text{ and } \log_e x = \ln x$$

### Product Property of Logarithms (p. 507)

$$\log_b mn = \log_b m + \log_b n$$

### Quotient Property of Logarithms (p. 507)

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

### Power Property of Logarithms (p. 507)

$$\log_b m^n = n \log_b m$$

### Change of Base (p. 508)

$$\log_c a = \frac{\log_b a}{\log_b c}$$

## Properties of Functions

### Operations on Functions (pp. 428, 430)

Let  $f$  and  $g$  be any two functions. A new function  $h$  can be defined using any of the following operations.

**Addition:**  $h(x) = f(x) + g(x)$

**Subtraction:**  $h(x) = f(x) - g(x)$

**Multiplication:**  $h(x) = f(x) \cdot g(x)$

**Division:**  $h(x) = \frac{f(x)}{g(x)}$

**Composition:**  $h(x) = g(f(x))$

For addition, subtraction, multiplication, and division, the domain of  $h$  consists of the  $x$ -values that are in the domains of both  $f$  and  $g$ . Additionally, the domain of the quotient does not include  $x$ -values for which  $g(x) = 0$ .

For composition, the domain of  $h$  is the set of all  $x$ -values such that  $x$  is in the domain of  $f$  and  $f(x)$  is in the domain of  $g$ .

### Inverse Functions (p. 438)

Functions  $f$  and  $g$  are inverses of each other provided:  
 $f(g(x)) = x$  and  $g(f(x)) = x$