## Properties

## Properties of Real Numbers

|  | Let $a, b$, and $c$ be real numbers. |  |
| :--- | :--- | :--- |
|  | Addition | Multiplication |
| Closure Property (p. 3) | $a+b$ is a real number. | $a b$ is a real number. |
| Commutative Property (p. 3) | $a+b=b+a$ | $a b=b a$ |
| Associative Property (p. 3) | $(a+b)+c=a+(b+c)$ | $(a b) c=a(b c)$ |
| Identity Property (p. 3) | $a+0=a, 0+a=a$ | $a \cdot 1=a, 1 \cdot a=a$ |
| Inverse Property (p. 3) | $a+(-a)=0$ | $a \cdot \frac{1}{a}=1, a \neq 0$ |
| Distributive Property (p. 3) | The distributive property involves both addition and multiplication:  <br>  $a(b+c)=a b+a c$ <br> Zero Product Property (p. 253) Let $A$ and $B$ be real numbers or algebraic expressions. <br>  If $A B=0$, then $A=0$ or $B=0$. |  |

## Properties of Matrices

Associative Property of Addition (p. 188)
Commutative Property of Addition (p. 188)
Distributive Property of Addition (p. 188)
Distributive Property of Subtraction (p. 188)
Associative Property of Matrix Multiplication (p. 197)
Left Distributive Property of Matrix Multiplication (p. 197)
Right Distributive Property of Matrix Multiplication (p. 197)
Associative Property of Scalar Multiplication (p. 197)
Multiplicative Identity (p. 210)

Inverse Matrices (p. 210)

Let $A, B$, and $C$ be matrices, and let $k$ be a scalar.

$$
\begin{aligned}
& (A+B)+C=A+(B+C) \\
& A+B=B+A \\
& k(A+B)=k A+k B \\
& k(A-B)=k A-k B \\
& (A B) C=A(B C) \\
& A(B+C)=A B+A C \\
& (A+B) C=A C+B C \\
& k(A B)=(k A) B=A(k B)
\end{aligned}
$$

An $n \times n$ matrix with l's on the main diagonal and 0's elsewhere is an identity matrix, denoted $I$. For any $n \times n$ matrix $A, A I=I A=A$.

If the determinant of an $n \times n$ matrix $A$ is nonzero, then $A$ has an inverse, denoted $A^{-1}$, such that $A A^{-1}=A^{-1} A=I$.

## Properties of Exponents

Let $a$ and $b$ be real numbers, and let $m$ and $n$ be integers.
$a^{m} \cdot a^{n}=a^{m+n}$
$\left(a^{m}\right)^{n}=a^{m n}$
$(a b)^{m}=a^{m} b^{m}$
$a^{-m}=\frac{1}{a^{m}}, a \neq 0$
$a^{0}=1, a \neq 0$
$\frac{a^{m}}{a^{n}}=a^{m-n}, a \neq 0$
$\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}, b \neq 0$

