

Formulas and Theorems from Algebra (continued)

<p>Discriminant of a general second-degree equation (p. 653)</p>	<p>Any conic can be described by a general second-degree equation in x and y: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The expression $B^2 - 4AC$ is the discriminant of the conic equation and can be used to identify it.</p> <table border="0" style="width: 100%;"> <thead> <tr> <th style="text-align: left;">Discriminant</th> <th style="text-align: left;">Type of Conic</th> </tr> </thead> <tbody> <tr> <td>$B^2 - 4AC < 0$, $B = 0$, and $A = C$</td> <td>Circle</td> </tr> <tr> <td>$B^2 - 4AC < 0$, and either $B \neq 0$ or $A \neq C$</td> <td>Ellipse</td> </tr> <tr> <td>$B^2 - 4AC = 0$</td> <td>Parabola</td> </tr> <tr> <td>$B^2 - 4AC > 0$</td> <td>Hyperbola</td> </tr> </tbody> </table> <p>If $B = 0$, each axis of the conic is horizontal or vertical.</p>	Discriminant	Type of Conic	$B^2 - 4AC < 0$, $B = 0$, and $A = C$	Circle	$B^2 - 4AC < 0$, and either $B \neq 0$ or $A \neq C$	Ellipse	$B^2 - 4AC = 0$	Parabola	$B^2 - 4AC > 0$	Hyperbola
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Formulas from Combinatorics

<p>Fundamental counting principle (p. 682)</p>	<p>If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events can occur is $m \cdot n$.</p>
<p>Permutations of n objects taken r at a time (p. 685)</p>	<p>The number of permutations of r objects taken from a group of n distinct objects is denoted by ${}_n P_r$ and is given by:</p> ${}_n P_r = \frac{n!}{(n-r)!}$
<p>Permutations with repetition (p. 685)</p>	<p>The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on is:</p> $\frac{n!}{s_1! \cdot s_2! \cdot \dots \cdot s_k!}$
<p>Combinations of n objects taken r at a time (p. 690)</p>	<p>The number of combinations of r objects taken from a group of n distinct objects is denoted by ${}_n C_r$ and is given by:</p> ${}_n C_r = \frac{n!}{(n-r)! \cdot r!}$
<p>Pascal's triangle (p. 692)</p>	<p>If you arrange the values of ${}_n C_r$ in a triangular pattern in which each row corresponds to a value of n, you get what is called Pascal's triangle.</p> $ \begin{array}{cccccc} & & & & & 1 \\ & & & & & & 1 \\ & & & & & 1 & 1 \\ & & & & & 1 & 2 & 1 \\ & & & & & 1 & 3 & 3 & 1 \\ & & & & & 1 & 4 & 6 & 4 & 1 \end{array} $ <p>The first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it.</p>
<p>Binomial theorem (p. 693)</p>	<p>The binomial expansion of $(a + b)^n$ for any positive integer n is:</p> $ \begin{aligned} (a + b)^n &= {}_n C_0 a^n b^0 + {}_n C_1 a^{n-1} b^1 + {}_n C_2 a^{n-2} b^2 + \dots + {}_n C_n a^0 b^n \\ &= \sum_{r=0}^n {}_n C_r a^{n-r} b^r \end{aligned} $