Formulas and Theorems from Algebra

Quadratic formula (p. 292)	The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a, b, and c are real numbers such that $a \neq 0$.
Discriminant of a quadratic equation (p. 294)	The expression $b^2 - 4ac$ is called the discriminant of the associated equation $ax^2 + bx + c = 0$. The value of the discriminant can be positive, zero, or negative, which corresponds to an equation having two real solutions, one real solution, or two imaginary solutions, respectively.
Special product patterns (p. 347)	**Sum and difference:** $(a + b)(a - b) = a^2 - b^2$ **Square of a binomial:** $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ **Cube of a binomial:** $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
Special factoring patterns (p. 354)	**Sum of two cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ **Difference of two cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Remainder theorem (p. 363)	If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.
Factor theorem (p. 364)	A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.
Rational zero theorem (p. 370)	If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational zero of f has this form: $\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$
Fundamental theorem of algebra (p. 379)	If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.
Corollary to the fundamental theorem of algebra (p. 379)	If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as 2 solutions, each solution repeated three times is counted as 3 solutions, and so on.
Complex conjugates theorem (p. 380)	If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.
Irrational conjugates theorem (p. 380)	Suppose f is a polynomial function with rational coefficients, and a and b are rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.
Descartes' rule of signs (p. 381)	Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with real coefficients. • The number of *positive real zeros* of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number. • The number of *negative real zeros* of f is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

TABLES