Logical Argument

TEKS G.3.C, G.3.E

A logical argument has two given statements, called **premises**, and a statement, called a **conclusion**, that follows from the premises. Below is an example.

Premise 1 If a triangle has a right angle, then it is a right triangle.
Premise 2 In $\triangle ABC$, $\angle B$ is a right angle.
Conclusion $\triangle ABC$ is a right triangle.

Letters are often used to represent the statements of a logical argument and to write a pattern for the argument. The table below gives five types of logical arguments. In the examples, p, q, and r represent the following statements.

p: a figure is a square q: a figure is a rectangle r: a figure is a parallelogram

Type of Argument	Pattern	Example
Direct Argument	If p is true, then q is true. p is true. Therefore, q is true.	If *ABCD* is a square, then it is a rectangle. *ABCD* is a square. Therefore, *ABCD* is a rectangle.
Indirect Argument	If p is true, then q is true. q is not true. Therefore, p is not true.	If *ABCD* is a square, then it is a rectangle. *ABCD* is not a rectangle. Therefore, *ABCD* is not a square.
Chain Rule	If p is true, then q is true. If q is true, then r is true. Therefore, if p, then r.	If *ABCD* is a square, then it is a rectangle. If *ABCD* is a rectangle, then it is a parallelogram. Therefore, if *ABCD* is a square, then it is a parallelogram.
Or Rule	p is true or q is true. p is not true. Therefore, q is true.	*ABCD* is a square or a rectangle. *ABCD* is not a square. Therefore, *ABCD* is a rectangle.
And Rule	p and q are not both true. q is true. Therefore, p is not true.	*ABCD* is not both a square and a rectangle. *ABCD* is a rectangle. Therefore, *ABCD* is not a square.

An argument that follows one of these patterns correctly has a **valid conclusion**.

EXAMPLE **State whether the conclusion is *valid* or *invalid*. If the conclusion is valid, name the type of logical argument used.**

a. If it is raining at noon, Peter's family will not have a picnic lunch. Peter's family had a picnic lunch. Therefore, it was not raining at noon.

▸ The conclusion is valid. This is an example of indirect argument.

b. If a triangle is equilateral, then it is an acute triangle. Triangle *XYZ* is an acute triangle. Therefore, triangle *XYZ* is equilateral.

▸ The conclusion is invalid.

c. If $x = 4$, then $2x - 7 = 1$. If $2x - 7 = 1$, then $2x = 8$. $x = 4$. Therefore, if $x = 4$, then $2x = 8$.

▸ The conclusion is valid. This is an example of the chain rule.

d. If it is at least 80°F outside today, you will go swimming. It is 85°F outside today. Therefore, you will go swimming.

▸ The conclusion is valid. This is an example of direct argument.